Mathematical modelling of batch and continuous bacterial leaching

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Abstract

A mathematical model of bacterial leaching reactors is presented. The model incorporates the population (number) balance approach for the modelling of particulate leaching, the balance for the bacterial cell number, which includes populations of bacteria on the mineral surface and in solution, and material balances describing solution reactants and products. The effects of particle-size distribution, particle kinetics, and the residence-time distribution are accounted for in the population balance. The bacterial balances account for the growth of bacteria in solution and on the mineral surface, and for the attachment to and detachment from the mineral surface. The model is solved for the case of batch leaching and for the case of continuous leaching. The batch model is shown to be a good description of batch leaching. Parameters obtained from batch data for the leaching of a pyritic ore are used to predict the performance of a continuous pilot plant. The model predicts results which compare very favourably with data from the continuous pilot plant.

1. Introduction

Bacterial oxidation of sulphide minerals is a well known phenomenon, and has been commercially implemented in dump-leaching processes for the recovery of copper and uranium. These dump-leaching processes are low cost operations, and little attempt has been made to optimize the parameters affecting microbial action.

Bacterial oxidation has also been utilized in the pretreatment of refractory gold ores. Gold is finely disseminated in the pyrite and arsenopyrite minerals in these ores, and bacterial oxidation of these minerals can improve the recovery of gold significantly. Processes have been evaluated to assess the commercial implementation of bacterial leaching as a pretreatment for refractory gold ores. Genmin has operated a commercial leaching plant since 1984 at Fairview Gold Mine, Barberton, South Africa. The plant employs continuous-stirred-tank reactors, and treats an ore containing arsenopyrite and pyrite [1]. A process has also been developed and piloted at Equity Silver Mine, British Columbia [2]. In addition, a number of feasibility studies have been performed [3, 4].

In order for bacterial leaching to compete successfully with other pretreatment processes for refractory ores, particularly with established technologies such as pressure leaching, it needs to be efficient. This requires the optimization of the parameters affecting the leaching reaction and the growth of bacteria.

Bacterial interactions are highly complex, and analysis is complicated by the presence of solids in leaching systems. In spite of the amount of research that has been performed, kinetic and process models of the bacterial leaching of ores are underdeveloped.

Mathematical models of leaching in continuous tank reactors (CSTR) in the absence of bacteria have received much attention from hydrometallurgists. The nature of the problem is more complex than that of batch leaching since the particle-size distribution changes owing to the leaching reaction and to the flow of material into and out of the tank reactor. The population-balance model has been developed to describe the behaviour of particles on reaction [5]. Sepulveda and Herbst [6] applied this model to leaching reactors, and Crundwell and Bryson [7] demonstrated that continuous plant data could be predicted from the batch kinetic data using this model.

The purpose of this paper is to propose a mathematical model of bacterial leaching that incorporates both knowledge of the bacterial phenomena and knowledge of modelling of leaching. The proposed model is based on the population balance.
2. Review of previous models of bacterial leaching

The factors influencing the bacterial oxidation of sulphides have been studied extensively, and several reviews of the literature have been published [2, 8–12].

The bacteria are known to attach to the mineral particles, and etch patterns form where the bacteria are located [13]. These bacteria are also able to oxidize ferrous ions in solution. The product of this reaction, ferric ions, is a powerful oxidizing agent which is also capable of oxidizing sulphide minerals.

Therefore, the bacteria are able to affect leaching both by attachment to the sulphide solids and by the production of ferric ions. This dual role of bacteria has resulted in one of the major debates surrounding bacterial leaching, that is, the debate concerning which mechanism dominates the bacterial leaching process.

As a result, bacterial interaction with sulphide minerals has often been separated into two processes. The direct mechanism is that in which bacteria attached to the surface of the particle oxidize the sulphide mineral, while the indirect mechanism is that in which bacteria oxidize ferrous ions, and the resulting ferric ions oxidize the sulphide mineral. That both mechanisms occur is undeniable; however, the conditions under which one or the other mechanism dominates the leaching process are not yet known.

It has proven to be difficult to distinguish between the direct and indirect mechanism experimentally. The products, and most probably the intermediates too, are the same in both cases. The oxidation of pyrite by both mechanisms is believed to occur in an active culture [14]. Recently, Espejo and Ruiz [15] have determined that the total activity associated with direct bacterial attack was between 1% and 10%. In contrast, Boogerd et al. [16] determined that only between 8% and 17% of pyrite was oxidized chemically at 45 °C.

Thus, kinetic models of bacterial leaching can be divided into two broad categories, reflecting the debate centred on the relative importance of the direct and indirect leaching mechanisms. The first category consists of those models which describe the extent of leaching, usually by shrinking-particle kinetics, and relate bacterial growth to this, and the second category consists of those models which describe the bacterial growth and attachment, usually by exponential kinetics, and relate the leaching to this.

Chaudhury et al. [17], Blancarte-Zurita et al. [18, 19], Chaudhury and Das [20] and Verbaan and Huberts [21] described bacterial leaching by the shrinking-particle model. The results of Chaudhury et al. [17] for the bacterial leaching of sphalerite are in agreement with the results obtained in the absence of bacteria [22]. No account was made in these models for the dependence of the reaction rate on the bacterial number.

Hansford and Drossou [23] and Southwood and Southwood [24] proposed a propagating-pore model as an alternative to the shrinking-particle model. However, it was not shown that the main mechanism of dissolution was by the development of pores.

The second category of the modelling of bacterial leaching is that which has concentrated on the description of bacterial growth.

Gormely et al. [25] assumed that bacterial attachment to the surface is a prerequisite for dissolution and bacterial growth. Their model for the continuous leaching of sphalerite was based on that of Erickson et al. [26], and described bacterial attachment to the surface of the mineral by a Langmuir-type adsorption isotherm. The rate of growth is proportional to the number of bacteria attached to the surface. No allowance is made for the change in surface area as a function of the extent of reaction.

Attempts to improve the model of Gormely et al. [25] have concentrated on incorporating the effect that the reduction in surface area has on the course of leaching [27] and allowing for the growth of bacteria in solution [28].

Kargi and Weissman [27] and Lizama and Suzuki [29] accounted for the change in the amount of pyrite present using a Monod-type dependence on the amount of pyrite present, even though this is a heterogeneous reaction. It is not clear how Monod kinetics, which are an extrapolation of Michaelis–Menten enzyme kinetics, could be extended to describe the dependence on a solid substrate.

Neuberg et al. [30] described a kinetic model for the operation of a percolation column. They described the dissolution kinetics by the shrinking-core model, and described the growth of bacteria by a model similar to that of Chang and Myerson [28] and Kargi and Weissman [27]. Bacterial growth is allowed in both the solution and on the mineral surface.

Asai et al. [31] and Konishi et al. [32] have recently proposed models that account for the
change in surface area during leaching. However, further improvements can possibly be made to their work; for instance, the leaching of sphalerite in the absence of bacteria was described by first-order homogeneous kinetics rather than the more acceptable shrinking-core kinetics.

For the direct mechanism to occur, bacteria need to attach to the surface of the mineral particles. Several authors have reported isotherms for bacterial attachment to sulphides [31, 33-35]. Basaran and Tuovinen [36] questioned the results of these studies, which were based on short-term experiments. They reported that 17%-34% of the cells were associated with the solids after 35 days. In contrast, complete association was reported by DiSpirito et al. [37] and Bagdigan and Myerson [38], whose experiments lasted 6 h and 1 h, respectively.

Pinches et al. [39], Hansford and Chapman [40] and Miller and Hansford [41] fitted the logistic equation [42, 43] to results for both batch and continuous experiments. The logistic equation describes the exponential growth and the stationary phase in batch growth curves. These authors assumed that the biomass was proportional to the fractional conversion of the pyrite. They compared the parameters obtained from the batch and the continuous experiments. The parameter which quantified the stationary phase and hence the maximum conversion was in good agreement for both batch and continuous experiments, simply because the reaction proceeded to the same extent in both cases. However, the maximum growth rate differed by a factor of between 1.6 and 2.7 in the work of Pinches et al. [39].

The derivation of a model for continuous bacterial leaching at steady state from the logistic equation by these authors may be criticized. Even if the particle-size distribution of the feed to a continuous reactor is narrow or the feed particles are of a single size, the size distribution in the reactor will be wider, since particles will spend different times in the reactor as a result of the residence-time distribution of the reactor.

None of the previous models of bacterial leaching in a continuous-stirred-tank reactor (CSTR) have accounted for the change in particle-size distribution due to flow of material into and out of the reactor and due to the shrinkage in particle size as a result of the leaching reaction.

3. Mathematical modelling of bacterial leaching

Leaching in the presence of bacteria consists of the processes of dissolution of the mineral and the growth of bacteria. These processes are related to each other through the ferrous and ferric couple, and through the direct bacterial oxidation of the mineral.

Mathematical models of leaching are well known [6, 7] and are presented here in terms of the population balance [5].

3.1. Growth of bacteria

Let us consider the biomass to be composed of indistinguishable cells all of the same physiological state. The biomass is divided into two populations: that which lives in solution, and that which lives on the mineral surface. The members of these populations are free to move between the two groups. The movement between groups is achieved by attachment and detachment from the mineral surface.

A balance on the bacterial biomass that is on the surface of the mineral particles gives (see Appendix A)

$$\frac{d(NV)}{dt} = Q((N_A)_f - (N_A)) + r_{ms}V + r_aV - r_dV$$ (1)

where $N$ is the number of bacteria on the surface per unit area, $V$ is the volume of the reactor, $A$ is the particle surface area per unit volume of slurry, $Q$ is the volumetric flow-rate, $r_{ms}$ is the rate of attachment of bacteria onto the mineral surface, and $r_d$ is the rate of detachment of the bacteria from the mineral surface. The subscript $f$ represents that quantity in the feed to the reactor. If it is assumed that the volume is constant, then

$$\frac{d(NA)}{dt} = \frac{1}{\tau} ((N_A)_f - (N_A)) + r_{ms} + r_a - r_d$$ (2)

where $\tau = V/Q$.

The rate of attachment is proportional to the unoccupied space on the surface, and is given by

$$r_a = k_a(N_{max}A - NA)$$ (3)

where $N_{max}$ represents the maximum number of bacteria on the particle per unit particle area, and $M$ is the number per unit volume living in solution.

The rate of detachment is proportional to the occupied space on the surface, and is given by

$$r_d = k_dNA$$ (4)

The rate of growth of bacteria present on the mineral surface is proportional to the number present on the surface and to the available surface area:

$$r_{ms} = k_{ms}NA(N_{max}A - NA)$$ (5)

A balance of the biomass in solution gives
\[
\frac{d(MV)}{dt} = Q(M_f - M) + r_s V + r_d V - r_a V
\]  
\hspace{1cm} \text{(6)}
\[
\text{and if } V \text{ is constant, then}
\]
\[
\frac{dM}{dt} = \frac{1}{\tau} (M_f - M) + r_s + r_d - r_a
\]  
\hspace{1cm} \text{(7)}

It has been assumed that the bacteria that live on the mineral surface utilize sulphide as their only substrate, and that those that live in solution utilize ferrous ions as their only substrate.

The rate of growth in the solution expressed per unit volume of slurry is given by a Monod expression:
\[
r_s = \frac{k_s M [\text{Fe}^{2+}]}{K_m + [\text{Fe}^{2+}]}
\]  
\hspace{1cm} \text{(8)}

This expression may be expanded to include effects such as product inhibition.

These equations must be solved in conjunction with the equations describing the leaching reaction which are given in Section 3.2, and with the ferrous and ferric mass balances which are given in Section 3.3.

### 3.2. Leaching of minerals

Leaching is a process that comprises a disperse phase, which is a collection of particles, and a continuous phase. The particles of the disperse phase have a distribution of properties, such as size, mineralogy and reactivity. The number balance recognizes that these particles are countable entities, and as such their numbers are conserved.

Let the size of the particles that are being leached be denoted by \( l \), where \( l \) is defined by the relationship that \( l^3 \) is equivalent to the volume of the particle. The area of the particle is given by \( c \), where \( c \) is a shape factor which is a constant.

If \( n(l)dl \) represents the number of particles per unit volume of reactor that have a size in the range \( l \) to \( l + dl \), then the number balance for a well mixed stirred-tank reactor can be given by [5]
\[
\frac{d[R(l)n(l, t)]}{dt} + \frac{dn(l, t)}{dt} = Qn(l, t) - Qn(l, t)
\]  
\hspace{1cm} \text{(9)}

The kinetics of the reaction are incorporated in the term \( R(l) \), which is the rate of shrinkage of a particle of size \( l \).

The solution of the number balance gives \( n(l, t) \), from which the conversion \( X \) can be obtained from the equation
\[
1 - X = \frac{\int_{0}^{l} l^3 n(l, t) \, dl}{\int_{0}^{l} l^3 n(l) \, dl}
\]  
\hspace{1cm} \text{(10)}

The kinetics of leaching are affected by the quantity of bacteria on the surface, and by the concentration of ferric ions in solution. We propose that the rate of shrinkage of a particle of size \( l \) is given by
\[
R(l) = \frac{dl}{dt} = - \frac{k_{al} N \phi l^2}{3 \rho l^2} (N_{\text{max}} \phi l^2 - N \phi l^2)
\]  
\hspace{1cm} \text{(11)}

where \( \rho \) is the molar density of the mineral, and \( \alpha \) is the reaction order with respect to the concentration of ferric ions \([\text{Fe}^{3+}] \). The reaction order with respect to the concentration of the ferrous ions is \(-\alpha\). This functional form of the dependence of the rate on the concentrations of ferric and ferrous ions in solution can be derived from the mixed-potential model for leaching [22, 44]. The value of \( \alpha \) has been found to be between 0.5 and 1.1 [44].

**Batch leaching of a narrow size distribution**

It is informative to consider the case in which particles all of the same initial size are leached in a batch reactor. In this case the solution of the partial differential equation (9) is trivial, namely all the particles that were in the reactor at time \( t = 0 \) are still in the reactor at time \( t \), unless \( t \) is greater than the time for complete conversion. Then, eqn. (10) becomes
\[
1 - X = l^3/L^3
\]  
\hspace{1cm} \text{(12)}

where \( L \) is the size of the particles at time \( t = 0 \). The differential equation describing the conversion may be obtained from eqn. (11) directly by the substitution of eqn. (12):
\[
\frac{dX}{dt} = \frac{k_{al} N A}{F_0} (N_{\text{max}} A - NA) + \frac{k_{al} A}{F_0} \left(\frac{[\text{Fe}^{3+}]}{[\text{Fe}^{2+}]}\right)^\alpha
\]  
\hspace{1cm} \text{(13)}

where \( F_0 \) is the total number of moles of mineral in the batch reactor at time \( t = 0 \).

This equation indicates that the rate of dissolution of the mineral is influenced by the two processes of (direct) bacterial leaching and of (indirect) chemical leaching. In addition, the form of the equation for the rate of shrinkage has been chosen so that
the rate of dissolution of the mineral by direct bacterial leaching is proportional to the rate of growth of the bacteria on the surface of the mineral particles.

Noting that \( A = A_0 (1 - X)^{2/3} \) and that \( A_0 / F_0 = \phi / \rho L \), eqn. (13) may be written as

\[
\frac{dX}{dt} = \frac{k_{dl} NA}{F_0} (N_{max} A_0 (1 - X)^{2/3} - NA)
+ \frac{k_{u} \phi}{\rho L} (1 - X)^{2/3} \left( \frac{[Fe^{3+}]}{[Fe^{2+}]} \right)^m
\]

(14)

3.3. Mass balances for ferrous and ferric ions in solution

The mass balance of the ferrous ions present in solution is

\[
\frac{d[Fe^{2+}]_s}{dt} = Q_s ([Fe^{2+}]_i - [Fe^{2+}])
= \frac{Y_s V}{Y} + a \varepsilon_u V + b \varepsilon_{dl} V
\]

(15)

and the mass balance of the ferric ions present in solution is

\[
\frac{d[Fe^{3+}]_s}{dt} = Q_s ([Fe^{3+}]_i - [Fe^{3+}])
+ \frac{Y_s V}{Y} - c \varepsilon_u V
\]

(16)

where \( \varepsilon_u \) and \( \varepsilon_{dl} \) are the extents of leaching due to the (indirect) chemical and (direct) bacterial mechanisms respectively, and \( a, b, c \), are stoichiometric coefficients.

The conversion \( X \) is calculated by solving eqns. (9) and (10) in conjunction with eqn. (11) describing the kinetics of particle shrinkage. The conversion \( X \) is related to the extents of reaction by

\[
X = \frac{\varepsilon_u + \varepsilon_{dl}}{FQ}
\]

(17)

where \( F \) is the molar concentration of mineral per unit volume of slurry to the reactor, and \( Q \) is the volumetric flow-rate to the reactor. The extents of reaction are related to each other by the kinetics of reaction:

\[
\frac{\varepsilon_{dl}}{\varepsilon_u + \varepsilon_{dl}} = \frac{k_{dl} NA (N_{max} A - NA)}{k_{dl} NA (N_{max} A - NA) + k_{u} A [Fe^{3+}]^m / [Fe^{+}]^m}
\]

(18)

The model presented is capable of describing both the batch and the continuous operations of bacterial leaching. The equations account for some of the complex phenomena that are involved during bacterial leaching, which include the leaching reactions and the growth of bacteria under conditions where a choice of substrate is present.

In order to illustrate the model, the equations are solved for the cases of batch leaching and continuous leaching, and compared with experimental data for both these cases.

4. Application of model to batch leaching

Konishi et al. [32] presented data on the batch leaching of a narrow size fraction of sphalerite (ZnS) particles. They performed detailed experiments to determine the rate of leaching in the absence of bacteria, the rates of bacterial attachment onto and detachment from the mineral surface, and the rates of bacterial leaching in the presence and absence of ferric ions in solution. They determined the course of mineral leaching by measuring the amount of mineral that was dissolved, and determined the growth of bacteria by measuring the bacterial number in solution. The sphalerite used by Konishi et al. [32] had a zinc content of 55.7% and an iron content of 3%. A narrow size fraction, +37–57 \( \mu \)m, was used for the experiments. This size fraction had a specific surface of 46.2 \( m^2 \) kg\(^{-1}\).

The equations described in this section are initial-value ordinary differential equations. These equations were solved by numerical integration using IMSL package IVPAG, using Gear's method. The parameters were obtained by minimizing the sum of squared errors between the data and the model solution.

4.1. Leaching in the absence of bacteria

Konishi et al. [32] performed experiments to determine the rate of chemical leaching. In this case, eqn. (14) becomes

\[
\frac{dX}{dt} = \frac{k_{dl} \phi}{\rho L} (1 - X)^{2/3} \left( \frac{[Fe^{3+}]}{[Fe^{2+}]} \right)^m
\]

(19)

This equation is coupled with the mass balance equations, given by

\[
[Fe^{3+}] = [Fe^{3+}]_0 - 2F_0 X
\]

(20)

\[
[Fe^{2+}] = [Fe^{2+}]_0 + 2F_0 X + \frac{0.03}{0.557} 65.39 \frac{0.557}{55.85} F_0 X
\]

(21)

The last term on the right-hand side of eqn. (21) represents the amount of iron leached from the sphalerite.

The experiments were performed using 10 g l\(^{-1}\) sphalerite, which means that \( F_0 = 0.08518 \) mol l\(^{-1}\).
The best fit of the model to the experimental data is shown in Fig. 1. The values of the parameters are
\[ \frac{k_d \phi}{\rho \ell} = 0.0103 \text{ day}^{-1} \quad \alpha = 1.013 \]
The parameter \( \alpha \) is related to the mechanism of dissolution, and has previously been found to have a value in the region of 0.5 for sphalerite [22]. It is not clear why the value of \( \alpha \) is close to unity for these data. The values of these parameters are used in Section 4.4 in the model of bacterial leaching in the presence of ferric ions.

4.2. Bacterial attachment and detachment
Konishi et al. [32] presented data for the bacterial number in solution for short times after the mineral particles had been added. The data indicate a decrease in bacterial number in solution due to the attachment of the bacteria to the surface of the mineral particles. In this case for short times, there is no growth, so that \( k_{\text{m,s}} = 0 \) in eqn. (2). This batch process is therefore described by
\[ \frac{d(N_A)}{dt} = k_a(N_{\text{max}} - N_A)M - k_d N_A \] (22)
This equation must be solved in conjunction with conservation of the bacterial number for this short period, given by
\[ M = M_0 - N_A \] (23)
Since there is no leaching, \( A = A_0 = 46.2 \text{ m}^2 \text{ kg}^{-1} \times 0.01 \text{ kg l}^{-1} \). The best fit is shown in Fig. 2, and the values of the parameters are
\[ N_{\text{max}}' = 0.8 \times 10^{12} \text{ cells m}^{-2} \]
\[ k_a = 0.113 \times 10^{-10} \text{ cell}^{-1} \text{ day}^{-1} \]
\[ k_d = 0.661 \times 10^8 \text{ day}^{-1} \]

4.3. Bacterial leaching in the absence of ferric ions
In this experiment, Konishi et al. [32] investigated the rate of leaching in the absence of ferric ions in solution. From the model given in Section 3, the results of this experiment are described by eqns. (24)–(26). Equation (1) becomes
\[ \frac{d(N_A)}{dt} = k_a(N_{\text{max}} - N_A)M - k_d N_A \] (24)
Since there are no ferrous ions in solution, there is no growth in solution, and eqn. (7) becomes
\[ \frac{dM}{dt} = -k_a(N_{\text{max}} - N_A)M + k_d N_A \] (25)

![Graph](image)

Fig. 1. Leaching of sphalerite by ferric ions in the absence of bacteria. The data are from Konishi et al. [32]. The lines are calculated from the model, eqns. (19)–(21).
Since there are no ferric ions in solution, there is
no chemical leaching, and eqn. (13) becomes
\[
\frac{dX}{dt} = \frac{k_{\text{dl}}/N A}{F_0} (N_{\text{max}} A - NA) \tag{26}
\]
Noting that \( A = A_0 (1 - X)^{2/3}, \) where \( A_0 = 0.462 \text{ m}^2 \text{ l}^{-1}, \) eqns. (24)–(26) can be solved for \( NA, M, \) and \( X. \) The best fit for the parameters \( k_{\text{ms}}, k_{\text{dl}} \) is shown in Fig. 3. Other parameters in the model, namely \( k_r, k_d \) and \( N'_{\text{max}}, \) are the same as those obtained in Section 4.2. The values of the parameters fitted here are
\[
k_{\text{ms}} = 0.124 \times 10^{-10} \text{ l cell}^{-1} \text{ day}^{-1}
\]
\[
k_{\text{dl}} = 0.419 \times 10^{-24} \text{ l cells}^{-2} \text{ mol}^{-1} \text{ day}^{-1}
\]
\[
N_{\text{max}} = 0.391 \times 10^{12} \text{ cells m}^{-2}
\]

4.4. Bacterial leaching in the presence of ferric ions

The significance of the addition of ferric ions to
the batch reactor during bacterial leaching of sphal-
erite is that the ferric ions are themselves capable
of leaching the sphalerite. This process is described
by eqns. (27)–(31).

Equation (2) becomes
\[
\frac{d(NA)}{dt} = k_{\text{ms}} NA(N_{\text{max}} A - NA) + k_{\text{a}} (N'_{\text{max}} A - NA)M - k_d NA \tag{27}
\]
Equation (7) becomes
\[
\frac{dM}{dt} = \frac{k_g M[Fe^{2+}]}{K_m + [Fe^{2+}]} - k_a (N_{\text{max}} A - NA)M + k_d NA \tag{28}
\]
Equation (16) becomes
\[
\frac{d[Fe^{3+}]}{dt} = \frac{k_g M[Fe^{2+}]}{Y(K_m + [Fe^{2+}])} - 2\epsilon \tag{29}
\]
Equation (13) becomes
\[
\frac{dX}{dt} = \frac{k_{\text{dl}}/N A}{F_0} (N_{\text{max}} A - NA) + \frac{k_{\text{dl}} / \rho l_r (1 - X)^{2/3}}{\left(\frac{[Fe^{3+}]}{[Fe^{2+}]^a}\right)} \tag{30}
\]
The mass balance on the total amount of iron in
solution gives
\[
[Fe^{2+}] = [Fe^{3+}]_0 - [Fe^{3+}] + \frac{0.03 \times 65.39}{0.557 \times 55.89} F_0 X \tag{31}
\]
The last term on the right-hand side of eqn. (31) represents the amount of iron leached from the sphalerite.

Noting that $A = A_0(1 - X)^{2/3}$, eqns. (27)–(31) can be solved for $NA$, $M$, $X$, $[Fe^{3+}]$, and $[Fe^{2+}]$. The values for the other parameters in the model, namely $k_0\phi/pL$, $\alpha$, $k_a$, $k_d$, $N_{max}'$, $k_{ds}$, $k_{ms}$, and $N_{max}$, were the same as those obtained in Sections 4.1, 4.2 and 4.3. The best fit for the parameters $k_s$, $K_m$ and $Y$ is shown in Figs. 4 and 5. The values of the parameters are
4. Leaching of sphalerite by bacteria in the presence of ferric ions. The data are from Konishi et al. [32]. The lines are calculated from the model, eqns. (27)-(31).

\[ K_m = 1.0 \text{ mol}^{-1} \]
\[ Y = 0.106 \times 10^{14} \text{ cells mol}^{-1} \]
\[ k_s = 22.3 \text{ day}^{-1} \]

4.5. Discussion
The mathematical model of bacterial leaching, which was presented in Section 3, has been solved for the case of batch leaching of a narrow size fraction of sphalerite. The close agreement between the experimental data and the model suggests that the model developed here is a good description of the various processes occurring during the bacterial leaching of sphalerite. The parameters of the model have been estimated from several independent experiments.

5. Application of the model to continuous leaching
In this section, the mathematical model is used to predict the performance of a continuous pilot plant. Batch leach tests are analysed using the model in order to obtain values of the parameters to be used in the model of the continuous plant.

Experimental data for the batch and continuous bacterial leaching of a pyrite–arsenopyrite concentrate have been presented by Miller [45].

Miller [45] performed batch tests using a pyrite–arsenopyrite concentrate obtained from the Fairview Mine, Barberton, South Africa, and reported the results of a series of pilot-plant trials. Miller [45] determined the extent of reaction by determining the conversion of sulphidic minerals. No attempt was made to determine the bacterial number in solution, or to determine the concentrations of ferric and ferrous ions in solution.

The batch leaching tests were performed in 0.3 m³ vessels using a mixed Thiobacillus culture and an initial solid-to-liquid ratio of 1:8. The pH was adjusted to 1.6 before the reaction was initiated by introducing the bacterial inoculum.

The pilot plant consisted of four stages: the primary stage had a volume of 10 m³ and the following three stages had volumes of 5 m³. The feed to the reactor had a solid-to-liquid ratio of 1:8. Miller [45] reported the size distribution on a mass basis for the feed to the continuous pilot-plant. This distribution on a number basis is modelled by the gamma distribution with a mean of 25 μm, and a variance of 8 μm.

5.1. Analysis of batch leaching tests
If it is assumed that only direct bacterial leaching occurs, then the following equations may be written. From eqn. (2), we obtain
\[
\frac{d(NA)}{dt} = k_{\text{ms}}NA(N_{\text{max}}A - NA)
\]

and from equation (13) we obtain

\[
\frac{dX}{dt} = \frac{k_{\text{al}}}{F_0} NA(N_{\text{max}}A - NA)
\]

Dividing the equations by one another, we obtain

\[
\frac{dX}{d(NA)} = \frac{k_{\text{al}}}{k_{\text{ms}}F_0}
\]

(34)

Integration of eqn. (34) gives the relationship between the bacterial number on the surface and the mineral conversion:

\[
NA = (NA)_0 + \frac{k_{\text{ms}}F_0}{k_{\text{al}}} X
\]

(35)

where \((NA)_0\) is the bacterial number on the surface of the mineral particles at time \(t = 0\). This relationship establishes a proportionality between the bacterial number and the moles of sulphidic mineral that have been consumed.

If it is assumed that the mineral surface is sparsely covered with bacteria, then \(NA \ll N_{\text{max}}A\). Substituting eqn. (35) into eqn. (33) we obtain the following expression for the batch leaching of the pyrite–arsenopyrite concentrate:

\[
\frac{dX}{dt} = \frac{k_{\text{al}}}{{\rho}L} \left( (NA)_0 + \frac{k_{\text{ms}}F_0}{k_{\text{al}}} X \right) (1 - X)^{2/3}
\]

(36)

This equation is solved by numerical integration using IMSL package IVPAG using Gear’s method. The parameters were obtained by minimizing the sum of squared errors between the data and the model solution.

The best fit is shown in Fig. 6, and the values of the parameters are

\[
\frac{k_{\text{al}}(NA)_0 N_{\text{max}}{\phi}}{\rho} = 1.074 \text{ m day}^{-1}
\]

\[
\frac{k_{\text{ms}}F_0}{k_{\text{al}}(NA)_0} = 14.79 \text{ (unitless)}
\]

These parameters are used in the model of continuous bacterial leaching to predict the performance of the pilot plant.

5.2. Prediction of continuous plant performance

The performance of a steady-state continuous leach vessel is described by the following number balance:

\[
V \frac{d[R(l)n(l)]}{dl} = Qn(l) - Qn(l)
\]

(37)

This is a first-order differential equation, the solution of which is found using the method of integrating factors. The solution is given by

\[
n(l) = \frac{1}{R(l)\tau} \exp\left(-T/\tau\right) \int_{0}^{t} \exp(T/\tau)n_{i}(l) \, dl
\]

(38)

where

\[
T = \int_{0}^{t} 1/R(l) \, dl
\]

From eqn. (11), and the values of the parameters estimated from the batch leaching experiments, the rate of shrinkage is given by

\[
R(l) = - \frac{k_{\text{al}} N A N_{\text{max}}{\phi}}{3\rho} = - \frac{1.074}{3} \frac{NA}{(NA)_0}
\]

(39)

Once the exit distribution from a particular tank has been obtained, the conversion is calculated from eqn. (10).

The basis for the model of bacterial growth is that the rate of growth of bacteria on the mineral surface is proportional to the rate of consumption of moles of pyrite. This means that the number of bacteria on the mineral surface in the continuous leach is described by

\[
NA = (NA)_0 + \frac{k_{\text{ms}}F_t}{k_{\text{al}}(NA)_0} X
\]

(40)

where \(F_t\) is the molar concentration of pyrite per unit volume in the feed to the pilot plant. For the experimental data reported by Miller [45], \(F_t\) is equal to \(F_0\).

Dividing eqn. (40) by \((NA)_0\) and substituting the value of the second parameter estimated from the batch data, we obtain

\[
\frac{NA}{(NA)_0} = 1 + 14.79X
\]

(41)

Equations (38) and (41) (together with associated eqns. (10) and (39)) are solved simultaneously to obtain the conversion in a stage. Then the exit distribution for one stage becomes the feed distribution for the following stage, and eqns. (38) and (40) are solved for that stage.

The conversion of sulphide may thus be calculated. Results are presented in Fig. 7 using the parameters obtained from the batch leaching experiments.
Fig. 6. Leaching of a pyritic ore by bacteria. The data are from Miller [45]. The lines are calculated from the model, eqn (36).

Fig. 7. Leaching of a pyritic ore by bacteria in a continuous pilot plant. The data are from Miller [45]. The lines are calculated from the model, eqns. (38) and (40).

5.3. Discussion

Figure 7 illustrates the prediction of the continuous plant operation based on parameters estimated from the batch leaching experiments. A comparison of the data from the pilot plant and the model prediction indicates that there is good agreement.
between the model and the plant data. This suggests that the model presented here has accounted for the major effects that are important in both batch and continuous bacterial leaching.

Previous models have not accounted for the changing particle-size distribution (due to reaction and flow in a continuous reactor). These models, for example those of Pinches et al. [39] and Hansford and Chapman [40], reported large differences between the parameters estimated from the batch and continuous experimental data. It is believed that the major reason for this is that the particle leaching kinetics and the changing size-distribution were not accounted for in these models.

These results demonstrate that data from batch experiments of bacterial leaching may be used to predict the continuous performance of a bacterial leaching plant or pilot plant.

6. Conclusions

Previous models have not accounted for the complex particulate phenomena that occur during leaching. In this work, the number balance, which describes the particulate phenomena during leaching, has been combined with the mass balances describing the bacterial population on the mineral surface and the bacterial population in solution. As a result, the model developed here accounts for the effects of both leaching and bacterial growth.

This model has been solved and has been shown to be a good description of the batch data for the leaching of a sphalerite concentrate.

The model has been used to obtain parameters for the leaching of a pyritic ore from the batch data, and these parameters have been used to predict the performance of the continuous steady-state leaching of the same ore. The data from the continuous plant and the a priori prediction of the plant performance are in good agreement.

References


Appendix A: Nomenclature

\( a, b, c \) stoichiometric coefficients

\( A \) total surface area of mineral particles per unit volume of slurry (m\(^2\) l\(^{-1}\))

\( F \) concentration of mineral to the reactor (mol l\(^{-1}\))

\( F_o \) concentration of mineral in the reactor at time \( t = 0 \) (mol l\(^{-1}\))

\( F_t \) concentration of mineral in the feed to the reactor (mol l\(^{-1}\))

\( k \) rate constant

\( l \) particle size (m)

\( L \) particle size at time \( t = 0 \) (m)

\( M \) number of bacteria in solution (cells l\(^{-1}\))

\( n(l, t) \) number density of particles of size \( l \) at time \( t \) (cells m\(^{-1}\) l\(^{-1}\))

\( N \) number of bacteria on mineral surface per unit area of mineral surface (cells m\(^{-2}\))

\( N_{max} \) maximum concentration of bacteria that can exist on the mineral surface (cells m\(^{-2}\))

\( Q \) flow-rate of slurry to the reactor (l day\(^{-1}\))

\( Q_s \) flow-rate of solution to the reactor (l day\(^{-1}\))

\( r_a \) rate of attachment (cells l\(^{-1}\) day\(^{-1}\))

\( r_d \) rate of detachment (cells l\(^{-1}\) day\(^{-1}\))

\( r_{ms} \) rate of growth of bacteria attached to the mineral (cells l\(^{-1}\) day\(^{-1}\))

\( R(l) \) rate of shrinkage of the mineral particle (m day\(^{-1}\))

\( t \) time (days)

\( V \) volume of the reactor (l)

\( V_o \) volume of solution in the reactor (l)

\( X \) conversion of mineral

Greek letters

\( \alpha \) reaction order (-)

\( \epsilon \) extent of reaction (mol l\(^{-1}\) day\(^{-1}\))

\( \rho \) molar density (mol m\(^{-3}\))
\[ \tau \quad \text{mean residence time (days)} \]
\[ \phi \quad \text{particle shape factor (-)} \]

Subscripts
- a: attachment
- d: detachment
- dl: direct leaching (bacterial)
- f: feed to reactor
- il: indirect leaching (chemical)
- ms: mineral surface
- 0: value at time \( t = 0 \)
- s: solution